

# Wideband Power Amplifiers Characterization by Undersampling: Zhu-Frank Sampling Theorem

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**Abstract** — In this paper a power amplifier is measured and characterized by the use of the generalized Zhu-Frank sampling theorem. A specially designed test-bed has been used allowing the bandwidth of the stimuli signal to be 100 MHz in the characterization process.

**Index Terms** — Communication system performance, modeling, nonlinear systems, power amplifiers, radio transmitters.

## I. INTRODUCTION

Future wireless communication systems are moving towards broad-band signals. Their frequencies will vary from couple of KHz to hundreds of Mhz. The power amplifier (PA) is a key radio component in any wireless communication system, and behavioral modeling of its input-output characteristics a developed research area.

Considering the PA as a nonlinear dynamic device, the spectral support of its outputs does not only cover the spectral support of the input, but also the adjacent channels. Relying on the Nyquist sampling theorem, sampling rates of several hundreds of MHz are required to sample the amplifier output according to the classical sampling theorem. From a hardware point of view, such requirement is difficult to satisfy. An alternative is the use of sub-sampling based on the seminal work by Zhu [1]. In short, Zhu's work implies that if there is a static invertable function that compresses the spectral support of an analog signal, it is sufficient to sample it with a speed corresponding to twice the bandwidth of the compressed signal. After ideal pulse-modulation to obtain the reconstructed signal, the inverse of the compressing function is applied to reconstructed data to obtain a full-band signal. The results of Zhu were later on generalized to nonlinear dynamic systems of certain classes by [2] and [3], and applied on a 3G WCDMA PA modeling in [4] and [5].

In the current work, we take the work of Wisell a step further by increasing the bandwidth of the signal to first 50 and then 96 MHz. This has been possible due to a specially designed test setup described in Section II [6]. The theory of the model identification is given in Section III and the results are presented in Section IV, followed by the conclusions in Section V.

## II. TEST SETUP

Thus, a specially designed test system has been designed for PA characterization based on ZFGST.

In order to master the wide bandwidth requirements, a specially designed test system has been used. It is characterized by an ultra wideband radio frequency (RF) front-end. It has been designed with an output frequency spectrum range of 20 - 1000 MHz and 14 dB gain with  $\pm 1.5$  dB amplitude variations. That is well enough for the subsequent 12-bit pipelined ADC intended for direct IF sampling that operates up to 210 MSPS conversion rate with analog bandwidth of 700 MHz.

## III. MODEL IDENTIFICATION

The model identification procedure has been described in [6]. Sampled input and output data records were measured at different time instants with the described measurement system. Cross-correlation and phase-compensation synchronization of the acquired time series was needed before model identification [7].

The models were identified by minimizing the mean square error (MSE) of the measured output and the model output. As model structure, the commonly used parallel Hammerstein (PH) model [8] is chosen. The model is defined by its nonlinear order  $P$  and the memory length  $M$ . Such a model is henceforth denoted  $\text{PH}(P,M)$ . The models were identified using the measured quantities of the input and output signals, known at the specific time instant, and formed to a model-specific regression matrix  $\Phi$ . The non-linear model behavior is absorbed by  $\Phi$ . The nonlinear model was described with the model predictor

$$\hat{y}(n) = \theta^T \Phi \quad (1)$$

which is linear in the parameters  $\theta$ . The least-squares estimation problem is then addressed as an overdetermined set of equations, linear in the parameters. Powerful and simple methods can then be used when determining those parameters.

#### IV. EXPERIMENTAL AND RESULTS

The tested amplifier is a 3G LDMOS PA with 52 dB gain and a maximum rated input power of 1 dBm. This PA is designed for use in the 2110-2170 MHz band with a peak gain variation of 0.5dB.

Due to the non-flat gain in the signal bandwidth, memory effects stronger than usual are expected. To obtain an accurate model this must be considered. The linear part of the PH model is simply a FIR-filter. With the variations within the signal bandwidth it is not necessarily true that the first coefficient of this filter is the largest coefficient. To check for possible "small" initial coefficients in the linear FIR-filter, a delay in the output signal was introduced with one sample at a time and identification was done for each such delay. The NMSE was then computed for comparison. In no case was more than 10 samples delay tested.

For the normal 3.84 MHz WCDMA signal the model with lowest normalized MSE (NMSE) was the PH(9,4) with a NMSE of -40.2 dB and an adjacent channel error power ratio (ACEPR) of -56.5 dB [9]. No additional delays were required to obtain the latter model. These results with the found model order and model errors are in line with the results from [7] for this particular PA.

The model order with lowest NMSE for the 50 MHz wide signal was the PH(9,7). In Fig. 1 the measured input, output and the model error for the PH(9,7) are shown. The necessary delay due to the wide bandwidth in this case was one sample yielding an improvement of 0.4 dB in NMSE as compared to using no extra delays.

For the 96 MHz wide signal the most suitable model based on NMSE was found to be a PH(9, 9) with a NMSE of -32.8 dB and a delay of 3 samples compared to what the synchronization found. Adding this delay improved the NMSE by 0.5 dB compared to no delay and same model order.

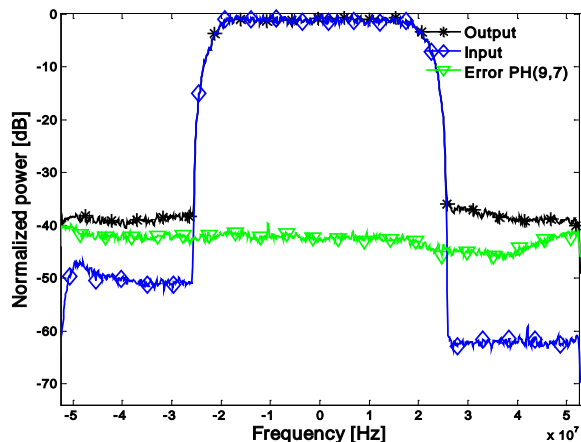


Fig. 1. The power spectra of the input, output and the model error of a PH(9,7) for the 50 MHz input signal.

#### VI. CONCLUSIONS

The ZFGST for the purpose of PA behavioral modeling was tested with different input signals of varying bandwidths, going from 3.84 MHz through 50 MHz to a 96 MHz wide input signal. Main difference in the models was the amount of required linear memory due to gain variations. For the wider signals, the normal cross-correlation based synchronization was no longer sufficient to find the optimal linear FIR-filter in the model. It was shown that introducing additional delays in the output signal as compared to the input signal improved the model performance with up to 0.5 dB for the same model order.

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