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Normative Positions within Norm-regulated Transition System Situations

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Abstract—In recent years, the study of norm-regulated multi-agent systems (MAS) has attracted a lot of attention. The Kanger-Lindahl theory of normative positions has great potential of serving as the logical foundation for normative systems for MAS, but its generality, which allows for great freedom when interpreting the theory, may also become a challenge for practical applications. As an important step towards a typology of interpretations of the theory, we study the application of normative positions in the context of a basic class of transition systems in which transitions are deterministic and associated with a single agent performing an act. By an interpretation of the types of normative positions in terms of permitting or prohibiting different types (with respect to some condition on a number of agents) of state transitions in this context, lexicons for two different systems of types of normative positions are suggested and discussed. It is demonstrated that both interpretations are natural and useful, depending on how the notion of agency is understood and whether a ‘system norms’ or an ‘agent-specific norms’ perspective is taken.

I. INTRODUCTION

The study of norm-regulated multi-agent systems covers areas such as the formal representation and implementation of normative systems as well as applications of norm-regulated MAS. One approach to the design of normative systems and the formulation of norms is the use of if-then-else rules or constraints on the states, and the transitions between states, of an agent or of the system as a whole. In many systems, the actions of an individual agent are naturally associated with transitions between different states of the system. As a consequence, the permission or prohibition of a specific action in such a system is connected to permissible or prohibited transitions between states of the system, and norms may then be formulated as restrictions on states and state transitions. The ‘agent-stranded transition systems’ framework by Craven and Sergot [1], [2] and the Ballroom system by Gaertner et al. [3] both serve as examples of this approach. Other approaches are algebraic or based on modal logics, like temporal or deontic logic. One example of the latter is the combination by Governatori et al. [4] of temporalised agency and temporalised normative positions, in the setting of Defeasible Logic. The DALMAS architecture for norm-regulated MAS [5] uses an algebraic approach, based on the Kanger-Lindahl system of one-agent types of normative positions¹, to the representation of normative systems.

¹The term ‘deontic positions’ has been suggested as a replacement for ‘normative positions’, but the latter term will be used here.

The Kanger-Lindahl theory of normative positions is widely discussed in the literature on deontic logic and MAS. Since the types of normative positions in the theory are mutually exclusive and exhaustive, the theory is well suited as a logical foundation for normative systems in a MAS context. From a theoretical point of view, the weak underlying assumptions makes the theory elegant and general, something which allows for great freedom when interpreting the theory. On the other hand, this becomes a challenge for practical applications. For example, the action operator Do/E_x , which in the theory of normative positions represents the notion of agency, can be understood in quite different ways, some of which may seem closer than others to the intuitive understanding of agency. Therefore, a kind of typology of interpretations of the theory of normative positions might be useful. It is hoped that this work may serve as a first step towards such a typology, by investigating how to apply the theory of one-agent normative positions in the context of a basic class of transition systems, in which transitions are deterministic and associated with a single agent performing an act. To study this class of systems, the notion of *norm-regulated transition system situations*, introduced previously by the present author, will be employed. In a norm-regulated transition system situation, the permission or prohibition of actions is connected to the permission or prohibition of different types (with respect to some condition on a number of agents in a state) of state transitions. By interpreting two different extended systems of one-agent types of normative positions in terms of permitting or prohibiting different transition types, a lexicon for each of these extended systems in the context of norm-regulated transition system situations will be obtained.²

The paper is structured as follows. Sect. I-A gives a brief introduction to the theory of normative positions, and Sect. I-B presents an extended system of normative positions suggested by Jones and Sergot. In Sect. I-C, the notion of norm-regulated transition system situations is presented, and an application to be used as a running example is introduced. Sect. II, the main contribution, discusses how to map Jones and Sergot’s extended system of normative positions, as well as another extension based on an observation by Odelstad, to the set of transition type prohibition operators within norm-regulated transition system situations. Sect. III briefly discusses possible

²Throughout the text, terms such as ‘interpretation’, ‘understanding’, ‘reading’ and ‘lexicon’ are used in an informal sense and more or less interchangeably.

TABLE I
ONE-AGENT TYPES OF NORMATIVE POSITIONS

T ₁	May Do(x, d) \wedge May[\neg Do(x, d) \wedge \neg Do($x, \neg d$)] \wedge May Do($x, \neg d$)
T ₂	May Do(x, d) \wedge May[\neg Do(x, d) \wedge \neg Do($x, \neg d$)] \wedge \neg May Do($x, \neg d$)
T ₃	May Do(x, d) \wedge \neg May[\neg Do(x, d) \wedge \neg Do($x, \neg d$)] \wedge May Do($x, \neg d$)
T ₄	\neg May Do(x, d) \wedge May[\neg Do(x, d) \wedge \neg Do($x, \neg d$)] \wedge May Do($x, \neg d$)
T ₅	May Do(x, d) \wedge \neg May[\neg Do(x, d) \wedge \neg Do($x, \neg d$)] \wedge \neg May Do($x, \neg d$)
T ₆	\neg May Do(x, d) \wedge May[\neg Do(x, d) \wedge \neg Do($x, \neg d$)] \wedge \neg May Do($x, \neg d$)
T ₇	\neg May Do(x, d) \wedge \neg May[\neg Do(x, d) \wedge \neg Do($x, \neg d$)] \wedge May Do($x, \neg d$)

TABLE II
TABLE 2 IN [12]

T ₁	$\left\{ \begin{array}{l} PE_a F \wedge PE_a \neg F \wedge P(F \wedge \neg E_a F) \wedge P(\neg F \wedge \neg E_a \neg F) \\ PE_a F \wedge PE_a \neg F \wedge \neg P(F \wedge \neg E_a F) \wedge P(\neg F \wedge \neg E_a \neg F) \\ PE_a F \wedge PE_a \neg F \wedge P(F \wedge \neg E_a F) \wedge \neg P(\neg F \wedge \neg E_a \neg F) \end{array} \right\}$
T ₂	$\left\{ \begin{array}{l} PE_a F \wedge \neg PE_a \neg F \wedge P(F \wedge \neg E_a F) \wedge P(\neg F \wedge \neg E_a \neg F) \\ PE_a F \wedge \neg PE_a \neg F \wedge \neg P(F \wedge \neg E_a F) \wedge P(\neg F \wedge \neg E_a \neg F) \\ PE_a F \wedge \neg PE_a \neg F \wedge P(F \wedge \neg E_a F) \wedge \neg P(\neg F \wedge \neg E_a \neg F) \end{array} \right\}$
T ₃	$\{PE_a F \wedge PE_a \neg F \wedge \neg PPass_a F\}$
T ₄	$\left\{ \begin{array}{l} \neg PE_a F \wedge PE_a \neg F \wedge P(F \wedge \neg E_a F) \wedge P(\neg F \wedge \neg E_a \neg F) \\ \neg PE_a F \wedge PE_a \neg F \wedge \neg P(F \wedge \neg E_a F) \wedge P(\neg F \wedge \neg E_a \neg F) \\ \neg PE_a F \wedge PE_a \neg F \wedge P(F \wedge \neg E_a F) \wedge \neg P(\neg F \wedge \neg E_a \neg F) \end{array} \right\}$
T ₅	$\{OE_a F\}$
T ₆	$\left\{ \begin{array}{l} OPass_a F \wedge OF \\ OPass_a F \wedge O\neg F \\ OPass_a F \wedge PF \wedge P\neg F \end{array} \right\}$
T ₇	$\{OE_a \neg F\}$

applications, and Sect. IV concludes and gives some ideas for future work.

A. One-Agent Types of Normative Positions

The Kanger-Lindahl theory of normative positions is based on Kanger's 'deontic action-logic'; see for example [6]. The theory, further developed by Lindahl in [7], contains three systems of types of normative positions. The simplest of these systems is a system of seven 'one-agent types' of normative positions, based on the logic of the action operator Do and the deontic operator Shall. Do(x, d) is read as ' x sees to it that d ' or ' x brings it about that d '. The logical properties assumed for Do is that it is the smallest system containing propositional logic, closed under logical equivalence and containing the axiom schema Do(x, d) \rightarrow d . The latter schema tries to capture the notion of *successful* action; if x 'sees to it' or 'brings about' that d , then d is indeed the case.

Each of the three statements (i) Do(x, d), (ii) Do($x, \neg d$) and (iii) \neg Do(x, d) \wedge \neg Do($x, \neg d$) implies the negation of each of the others, and the disjunction of all three is a tautology. Each of (i) - (iii) can be prefixed with either May or \neg May, where May F is defined as \neg Shall $\neg F$, and basic conjunctions containing one statement from each such pair are formed. By iterated construction of basic conjunctions, a set of eight conjunctions is obtained. One such 'maxi-conjunction' is self-contradictory, the other seven are listed in Table I.

Some further extensions of the systems of normative positions has been suggested by Jones and Sergot; see Sect. I-B. They have explored some applications of the theory within computer science, and discussed some of its limitations in this setting. Lindahl and Odelstad [8], [9] have combined the theory of normative positions with an algebraic approach to normative systems. DALMAS [5] is an abstract architecture for a class of multi-agent systems, regulated by normative systems based on this approach. A general-level Java/Prolog implementation of the DALMAS architecture has been developed [10], [11], to facilitate the implementation of specific systems.

B. An Extended Set of Types of Normative Positions

Jones and Sergot [13], [12] have generalised and further developed the Kanger-Lindahl theory of normative position. Using a method suitable for automation, they perform a similar analysis as Lindahl. First, a set of 'act positions'

- 1) $E_x F$
- 2) $E_x \neg F$
- 3) $\neg E_x F \wedge \neg E_x \neg F$

is generated from the scheme $\|\pm E_x \pm F\|$.³ Using O for Shall and P for May, each of the three logically consistent act positions is then prefixed with $\pm O \pm$, which yields a set of 64 maxi-conjunctions. Assuming the same logical properties of the O and E_x operators as in Lindahl's analysis, viz., the axiom of successful action, E.T., and closure under logical equivalence, E.RE., 57 of the 64 conjunctions are internally inconsistent. The remaining seven are precisely Lindahl's seven one-agent types of normative positions:

$$\|\pm O \pm \|\pm E_x \pm F\|\| = \|\pm P \|\pm E_x \pm F\|\| \quad (1)$$

For weaker logics this equality does not hold. The consequences of alternative logics of O or E_x will not be explored here. Instead, let us consider Sergot's idea to perform a refined analysis, based on the set of four 'cumulative fact/act positions'

- 1) $E_x F$
- 2) $E_x \neg F$
- 3) $F \wedge \neg E_x F$
- 4) $\neg F \wedge \neg E_x \neg F$

which is generated from the scheme $\|\pm E_x \pm F\| \cdot \|\pm F\|$. As before, these conjunctions are then prefixed by $\pm O \pm$:

³The operator E_x corresponds to the Kanger-Lindahl Do operator. $\|\pm E_x \pm F\|$ stands for the set of *maxi-conjunctions* of $\pm E_x \pm F$, i.e., the maximal consistent conjunctions of expressions of the form $\pm E_x \pm F$, where \pm stands for the two possibilities of affirmation and negation. This notation was suggested by Makinson [14].

$$\|\pm O \pm \|\pm E_x \pm F\| \cdot \|\pm F\|\| \quad (2)$$

This analysis yields a set of 15 logically consistent conjunctions, shown in Table 2 in [12, p. 375]. (The table is reiterated here as Table II. Note that $\text{Pass}_a F$ is used as an abbreviation for $\neg E_a F \wedge \neg E_a \neg F$.)

Lindahl's T_3 , T_5 and T_7 are identical to three of Sergot's 'normative act positions', while each of the other four types are logically equivalent to a disjunction of three conjunctions, as shown in the table.

C. Previous Work: Norm-regulated Transition System Situations

The notion of a norm-regulated transition system situation was originally presented in [15]. A transition system situation is intended to represent, for example, a 'snapshot' of a labelled transition system (LTS) in which each transition fulfills these criteria.

The normative framework of a norm-regulated transition system situation uses an algebraic representation of conditional norms and is based on a systematic exploration of the possible types, with respect to some state of affairs $d(x_1, \dots, x_n)$, of state transitions. A *transition system situation* is an ordered 5-tuple $\mathbf{S} = \langle x, s, A, \Omega, S \rangle$ characterised by a set of states S , a state s , an agent-set $\Omega = \{x_1, \dots, x_n\}$, the acting ('moving') agent x , and an action-set $A = \{a_1, \dots, a_m\}$. In this setting, a may be regarded as a function such that $a(x, s) = s^+$ means that s^+ is the resulting state when x performs act a in state s . In the following, the abbreviation s^+ will be used for $a(x, s)$ when there is no need for an explicit reference to the action a and the acting agent x . It is assumed that the action by the acting agent is deterministic and is performed asynchronously, i.e., that there is no simultaneous action by other agents (including the 'environment', which may be regarded as a special kind of agent). Furthermore, we assume that a ν -ary condition d is true or false on ν agents $x_1, \dots, x_\nu \in \Omega$ in s , where Ω is a set of agents associated with s ; this will be written $d(x_1, \dots, x_\nu; s)$. To facilitate the presentation, X_ν will often be used as an abbreviation for the argument sequence x_1, \dots, x_ν .⁴ Note that s may represent an arbitrary state in an LTS, and S is the set of states reachable from s by all transitions $x:a$, $a \in A$.

Now consider the transition from a state s to a following state s^+ , and focus on the state of affairs $d(X_\nu)$. With regard to $d(X_\nu)$, there are four possible alternatives for the transition from s to s^+ , since in s as well as in s^+ , $d(X_\nu)$ or $\neg d(X_\nu)$ could hold⁵:

$$\text{I. } d(X_\nu; s) \text{ and } d(X_\nu; s^+)$$

⁴In the special case when the sequence of agents is empty, i.e. $\nu = 0$, d represents a proposition which is true or false in s .

⁵Negations d' of conditions can be formed in the following way: $d'(X_\nu)$ iff $\neg d(X_\nu)$. Conjunctions ($c \wedge d$) and disjunctions ($c \vee d$) may be formed in a similar way; hence, it is possible to construct Boolean algebras of conditions.

- II. $\neg d(X_\nu; s)$ and $d(X_\nu; s^+)$
- III. $d(X_\nu; s)$ and $\neg d(X_\nu; s^+)$
- IV. $\neg d(X_\nu; s)$ and $\neg d(X_\nu; s^+)$

Each alternative represents a basic type of transition with regard to the state of affairs $d(X_\nu)$; we say that $\{\text{I, II, III, IV}\}$ is the set of *basic transition types* with regard to $d(X_\nu)$. The assignment of different types (with regard to some state of affairs) to state transitions closely resembles Sergot's ideas in [2].⁶ The main contribution here is that the types are systematically explored, and that the state of affairs is a condition which is true or false of a number of agents, not merely a proposition. Another notable difference is that, in Sergot's framework, the agent is said to bring it about that a transition has a certain type, rather than bringing about a certain state of affairs.

Let the *situation* $\langle x, s \rangle$ be characterised by the moving agent x and the state s in a transition system situation \mathbf{S} . To be able to determine the type, with respect to $d(X_\nu)$, of the transition represented by action a performed by agent $x_{\nu+1}$ in $\langle x, s \rangle$,⁷ we define a 'basic transition type operator' B_j^a , $j \in \{\text{I, II, III, IV}\}$, such that the $\nu + 1$ -ary 'transition type condition' $B_j^a d(X_\nu, x_{\nu+1}; x, s)$ indicates whether or not, in the situation $\langle x, s \rangle$, the event $x_{\nu+1}:a$ (representing a being performed by $x_{\nu+1}$) has basic transition type j with regard to $d(X_\nu)$: For all ν -ary conditions d and for all agents $X_\nu, x_{\nu+1}$, all acts a and all situations $\langle x, s \rangle$,

- I. $B_{\text{I}}^a d(X_\nu, x_{\nu+1}; x, s)$ iff $d(X_\nu; s) \wedge d(X_\nu; a(x_{\nu+1}, s))$
- II. $B_{\text{II}}^a d(X_\nu, x_{\nu+1}; x, s)$ iff $\neg d(X_\nu; s) \wedge d(X_\nu; a(x_{\nu+1}, s))$
- III. $B_{\text{III}}^a d(X_\nu, x_{\nu+1}; x, s)$ iff $d(X_\nu; s) \wedge \neg d(X_\nu; a(x_{\nu+1}, s))$
- IV. $B_{\text{IV}}^a d(X_\nu, x_{\nu+1}; x, s)$ iff $\neg d(X_\nu; s) \wedge \neg d(X_\nu; a(x_{\nu+1}, s))$

A *norm-regulated transition system situation* is represented by an ordered pair $\langle \mathbf{S}, \mathcal{N} \rangle$ where $\mathbf{S} = \langle x, s, A, \Omega, S \rangle$ is a transition system situation and \mathcal{N} is a normative system. It is assumed that norms apply to an individual agent $x_{\nu+1}$ in a state s . A norm in \mathcal{N} is represented by an ordered pair $\langle g, c \rangle$, where the (descriptive) condition g on a situation $\langle x, s \rangle$ is the *ground* of the norm and the (normative) condition c on $\langle x, s \rangle$ is its *consequence*. (See, e.g., [5].) Let us define a set of 'transition type prohibition operators' P_k , $k \in \mathcal{P}(\{\text{I, II, III, IV}\})$, where $\mathcal{P}(S)$ is the power set of S , such that $P_k d(X_\nu, x_{\nu+1}; x, s)$ indicates that the basic transition types (with respect to the state of affairs $d(X_\nu)$) in k are prohibited, and a set of corresponding 'transition type operators' C_k^a , such that $C_k^a d(X_\nu, x_{\nu+1}; x, s)$ iff the transition from s to $a(x_{\nu+1}, s)$ has any of the basic transition types in k with respect to $d(X_\nu)$.⁸ Then, for example, $\langle c, P_k d \rangle$ represents the sentence

⁶In the vein of [2], I could be written $0:d(X_\nu) \wedge 1:d(X_\nu)$, II could be written $0:\neg d(X_\nu) \wedge 1:d(X_\nu)$, and similarly for III and IV.

⁷Note that, in most systems, $x_{\nu+1} = x$. See [15] for an explanation.

⁸That is, P_k and C_k^a transform ν -ary conditions d on a state s to $\nu + 1$ -ary conditions $P_k d$ and $C_k^a d$ on a situation $\langle x, s \rangle$.

TABLE III
TABLE 1 IN [15]: POSSIBLE COMBINATIONS OF BASIC TRANSITION TYPES

	I	II	III	IV	
	-	-	-	-	-
$C_{\{IV\}}^a$	-	-	-	X	$\neg d(X_\nu; s) \wedge \neg d(X_\nu; s^+)$
$C_{\{III\}}^a$	-	-	X	-	$d(X_\nu; s) \wedge \neg d(X_\nu; s^+)$
$C_{\{III,IV\}}^a$	-	-	X	X	$\neg d(X_\nu; s^+)$
$C_{\{II\}}^a$	-	X	-	-	$\neg d(X_\nu; s) \wedge d(X_\nu; s^+)$
$C_{\{II,IV\}}^a$	-	X	-	X	$\neg d(X_\nu; s)$
$C_{\{II,III\}}^a$	-	X	X	-	$\neg(d(X_\nu; s) \leftrightarrow d(X_\nu; s^+))$
$C_{\{II,III,IV\}}^a$	-	X	X	X	$\neg d(X_\nu; s) \vee \neg d(X_\nu; s^+)$
$C_{\{I\}}^a$	X	-	-	-	$d(X_\nu; s) \wedge d(X_\nu; s^+)$
$C_{\{I,IV\}}^a$	X	-	-	X	$d(X_\nu; s) \leftrightarrow d(X_\nu; s^+)$
$C_{\{I,III\}}^a$	X	-	X	-	$d(X_\nu; s)$
$C_{\{I,III,IV\}}^a$	X	-	X	X	$d(X_\nu; s) \vee \neg d(X_\nu; s^+)$
$C_{\{I,II\}}^a$	X	X	-	-	$d(X_\nu; s^+)$
$C_{\{I,II,IV\}}^a$	X	X	-	X	$\neg d(X_\nu; s) \vee d(X_\nu; s^+)$
$C_{\{I,II,III\}}^a$	X	X	X	-	$d(X_\nu; s) \vee d(X_\nu; s^+)$
$C_{\{I,II,III,IV\}}^a$	X	X	X	X	\top

$$\forall x_1, x_2, \dots, x_\nu, x_{\nu+1} \in \Omega : c(x_1, x_2, \dots, x_p, x_{\nu+1}; x, s) \rightarrow P_k d(x_1, x_2, \dots, x_q, x_{\nu+1}; x, s)$$

where Ω is the set of agents, $x_{\nu+1}$ is the agent to which the norm applies, x is the acting agent in the situation $\langle x, s \rangle$, and $\nu = \max(p, q)$. If the condition specified by the ground of a norm is true in some situation, then the (normative) consequence of the norm is in effect in that situation. If the normative system contains a norm whose ground holds in the situation $\langle x, s \rangle$ and whose consequence prohibits the type of transition represented by the event $x_{\nu+1}:a$, then action a is prohibited for $x_{\nu+1}$ in $\langle x, s \rangle$:

Prohibited $_{x,s}(x_{\nu+1}, a)$ according to \mathcal{N}
if there exists a condition c and a condition d
and a $k \in \mathcal{P}(\{I, II, III, IV\})$,
such that $\langle c, P_k d \rangle$ is a norm in \mathcal{N} ,
and there exist $x_1, \dots, x_\nu, x_{\nu+1}$ such that
 $c(x_1, \dots, x_p, x_{\nu+1}; x, s) \ \& \ C_k^a d(x_1, \dots, x_q, x_{\nu+1}; x, s)$,
where $\nu = \max(p, q)$.

Hence, if $c(x_1, \dots, x_p, x_{\nu+1}; x, s)$ for some sequence of agents $x_1, \dots, x_q, x_{\nu+1}$, then the normative condition $P_k d(x_1, \dots, x_q, x_{\nu+1}; x, s)$ is ‘in effect’. Thus, if $C_k^a d(x_1, \dots, x_q, x_{\nu+1}; x, s)$ holds, then a is prohibited for $x_{\nu+1}$ in s . For example,

if $P_{\{I,IV\}} d$, then a is prohibited for $x_{\nu+1}$ in s
if $C_{\{I,IV\}}^a d(x_1, \dots, x_q, x_{\nu+1}; x, s)$
iff $B_I^a d(X_\nu, x_{\nu+1}; x, s)$ or $B_{IV}^a d(X_\nu, x_{\nu+1}; x, s)$.

The set of operators C_k^a is derived from the freely generated Boolean algebra over $d(x_1, \dots, x_\nu; s)$ and $d(x_1, \dots, x_\nu; a(x, s))$; see Table 1 in [15] (reiterated here as Table III).

1) *An example: ownership to an estate*: Let us construct an example of a norm-regulated transition system situation.

Although, admittedly, an oversimplification in many ways, the example is intended to capture some of the flavour of the ‘Ownership of an estate’ example employed in [9, Sect. 4.4.1]. Let us imagine a tiny world consisting of two neighbouring estates, Whiteacre (numbered 1) and Blackacre (2). The world is populated by the three agents Alice, Bob and Charlie, and each estate is owned by one of these agents. Furthermore, each estate contains a main building that can be painted white (W) or black (B), and can be surrounded by a fence.⁹ To keep things simple, there are only four actions available for the acting agent: *do nothing*, *paint the main building on Whiteacre white*, *paint the main building on Whiteacre black*, and *erect a fence that goes around both estates*. A state of this world can, e.g., be represented by a set consisting of two tuples $\langle E, O, P, F \rangle$ where $E \in \{1, 2\}$, $O \in \{Alice, Bob, Charlie\}$, $P \in \{W, B\}$, and $F \in \{Yes, No\}$. The intended meaning of, say, $\langle 1, Alice, W, No \rangle$ is that Whiteacre is owned by Alice, its main building is painted white, and it is not surrounded by a fence.

Let $\Omega = \{Alice, Bob, Charlie\}$ and let $A = \{n, p_{1,W}, p_{1,B}, e_{1,2}\}$ where n stands for ‘do nothing’, $p_{1,c}$ stands for ‘paint the main building on Whiteacre in the colour c ’, and $e_{1,2}$ stands for ‘erect a fence that goes around Whiteacre and Blackacre’. Define n such that $n(x, s) = s$, meaning that the result of x ’s doing nothing results in no change of state. Define p such that $p_{1,c}(x, \{\langle 1, o_1, c_1, f_1 \rangle, \langle 2, o_2, c_2, f_2 \rangle\}) = \{\langle 1, o_1, c, f_1 \rangle, \langle 2, o_2, c_2, f_2 \rangle\}$, i.e., no matter the current colour (c_1) of the main building on Whiteacre, the new colour will be c . All other parameters will be unaffected. Finally, define e such that $e_{1,2}(x, \{\langle 1, o_1, c_1, f_1 \rangle, \langle 2, o_2, c_2, f_2 \rangle\}) =$

⁹In the original example, each estate may also contain cows belonging to the estate itself, and possibly also stray cows from the neighbouring estate. This aspect of the example is not modeled here.

$\{ \langle 1, o_1, c_1, Yes \rangle, \langle 2, o_2, c_2, Yes \rangle \}$. In other words, no matter whether or not there is a fence around one or both estates, the effect when x performs $e_{1,2}$ is that both estates become surrounded by a fence. Note that these simple definitions fail to capture many real-world details; for example, the effect of erecting a fence around an already fenced estate will be the same as erecting a fence around a non-fenced estate.

We may now select one of the agents as the acting agent, and form a transition system situation S . For example, let $s = \{ \langle 1, Alice, W, No \rangle, \langle 2, Bob, B, No \rangle \}$, and let $S = \langle Alice, s, A, \Omega, S \rangle$. This means that Whiteacre is owned by Alice and is not surrounded by a fence, and Blackacre is owned by Alice and is not surrounded by a fence. Note that since Alice is the acting agent, S is implicitly defined by Alice's performing each of the actions in A in state s .

We will now create an adapted version of the normative system suggested in [9, Sect. 4.4.1]. Let o_1 and o_2 denote the unary conditions of being owner to Whiteacre and Blackacre, respectively. These conditions belong to the set of potential (descriptive) grounds of norms in the normative system. Furthermore, let w_1 and w_2 denote the 0-ary conditions of the main building on Whiteacre (resp., Blackacre) being painted white, and let $f_{1,2}$ denote the 0-ary condition of there being a fence going around both estates. These are the descriptive conditions that, by combining these with type-operators T_i and forming boolean combinations of these expressions, make up the set of potential (normative) consequences of norms. Note the subtle difference between the formulation of the last condition ('there being a fence ...') and the corresponding formulation in [9, Sect. 4.4.1]: 'erecting a fence ...'. Intuitively, their intended meaning is quite different, but both of these kinds of conditions are represented by the same kind of formalism in the theory of normative positions. A detailed discussion of these matters is, however, beyond the scope of this paper.

The following normative system is suggested:

- 1) $\langle o_1 \wedge o_2, T_1 w_1 \wedge T_1 w_2 \wedge T_1 f_{1,2} \rangle$
- 2) $\langle o_1 \wedge o'_2, T_1 w_1 \wedge T_6 w_2 \wedge T_4 f_{1,2} \rangle$
- 3) $\langle o'_1 \wedge o_2, T_6 w_1 \wedge T_1 w_2 \wedge T_4 f_{1,2} \rangle$
- 4) $\langle o'_1 \wedge o'_2, T_6 w_1 \wedge T_6 w_2 \wedge T_6 f_{1,2} \rangle$

To exemplify, $o_1 \wedge o_2$ means being the owner of both estates. This condition is a ground for $T_1 w_1 \wedge T_1 w_2 \wedge T_1 f_{1,2}$, which is the normative position-condition denoting full freedom with regard to painting the two buildings, and erecting a surrounding fence. In contrast, $o_1 \wedge o'_2$ means owning Whiteacre but not Blackacre; this condition is ground for $T_1 w_1 \wedge T_6 w_2 \wedge T_4 f_{1,2}$. This is intended to grant full freedom regarding the painting of building on Whiteacre, and no freedom to bring about or prevent painting of building on Blackacre. Furthermore, it is intended to grant freedom to prevent erection of a fence surrounding the estates and freedom to be 'passive' about the matter, but no freedom to bring about the fence's being erected. In Sect. II-2, we will return to this

example to suggest an interpretation in terms of prohibition of state transition types.

II. MAPPING NORMATIVE POSITIONS TO TRANSITION TYPE PROHIBITION OPERATORS

It is interesting to note that there are 15 conjunctions in Sergot's set of 'normative act positions' (see Sect. I-B), and also 15 rows in Table III (if we disregard the last row that prohibits all state transitions). Therefore, an interpretation of E_x that lets us identify each of the 'normative act positions' with one of the rows in the table comes natural in a norm-regulated transition system situation context. One way of reading $E_x F$ is ' x brings it about that F ', or even ' x that brings it about that F '; if F would be (or become) the case anyway, without intervention from the agent x , then it is not the case that it is x that brings it about that F .¹⁰ Under this interpretation it is reasonable to identify $E_x F$ with a state transition of type II in a norm-regulated transition system situation.¹¹ Consequently, a natural understanding of $\neg P E_x F$ in this context is that II be disallowed. Similar arguments may be given for $\neg P E_x \neg F$, $\neg P (\neg E_x F \wedge F)$ and $\neg P (\neg E_x \neg F \wedge \neg F)$. To summarise the discussion in [17, pp. 17f], the following principles can be assumed:

- 1) $\neg P E_x F$ implies that II be disallowed.
- 2) $\neg P E_x \neg F$ implies that III be disallowed.
- 3) $\neg P (\neg E_x F \wedge F)$ implies that I be disallowed.
- 4) $\neg P (\neg E_x \neg F \wedge \neg F)$ implies that IV be disallowed.

Using these principles, we identify Sergot's 'normative act positions' with a corresponding row in Table III. The result is shown in Table IV, with the rows ordered as in Sergot's Table 2.¹² The table shows a straightforward interpretation of Sergot's 'normative act positions' in terms of permissible and prohibited state transitions in the context of norm-regulated transition system situations. The immediate appeal of this interpretation is that it is both simple and intuitive. One might therefore say that the problem of finding a natural understanding of the Kanger-Lindahl theory of normative positions in the context of norm-regulated transition system situations is solved. So why not settle for this?

The answer may be a matter of different perspectives of norms and normative systems. If, for example, we are interested in formulating *system norms*, i.e., norms which "... express a system designer's point of view of what system states and transitions are legal, permitted, desirable, and so on" [2, p. 2], then the suggested understanding of normative act positions within the context of norm-regulated transition system situations is reasonable. On the other hand, if we talk about *agent-specific norms*, i.e., norms that apply to an individual agent in a specific situation to regulate which actions the agent may or may not perform, it can be argued

¹⁰See, e.g., [16]. For a further discussion of different ways of understanding the agency operator Do/E_x , see Sect. 3.1 in [17].

¹¹Clearly, this is consistent with the assumption that $E_x F \rightarrow F$.

¹²I.e., Table II in this paper. In order to adhere to the notation in [15], F is represented by $d(x_1, \dots, x_\nu)$. With explicit reference to states, we write $d(X_\nu; s)$ or $d(X_\nu; a(x, s))$, and so on.

TABLE IV
POSSIBLE INTERPRETATION OF SERGOT'S NORMATIVE ACT POSITIONS

	I: $\neg E_x F \wedge F$	II: $E_x F$	III: $E_x \neg F$	IV: $\neg E_x \neg F \wedge \neg F$	<i>Prohibited_a(x, s)</i> if
T _{1a}	-	-	-	-	-
T _{1b}	X	-	-	-	$d(X_\nu; s) \wedge d(X_\nu; a(x, s))$
T _{1c}	-	-	-	X	$\neg d(X_\nu; s) \wedge \neg d(X_\nu; a(x, s))$
T _{2a}	-	-	X	-	$d(X_\nu; s) \wedge \neg d(X_\nu; a(x, s))$
T _{2b}	X	-	X	-	$d(X_\nu; s)$
T _{2c}	-	-	X	X	$\neg d(X_\nu; a(x, s))$
T ₃	X	-	-	X	$d(X_\nu; s) \leftrightarrow d(X_\nu; a(x, s))$
T _{4a}	-	X	-	-	$\neg d(X_\nu; s) \wedge d(X_\nu; a(x, s))$
T _{4b}	X	X	-	-	$d(X_\nu; a(x, s))$
T _{4c}	-	X	-	X	$\neg d(X_\nu; s)$
T ₅	X	-	X	X	$d(X_\nu; s) \vee \neg d(X_\nu; a(x, s))$
T _{6a}	-	X	X	X	$\neg d(X_\nu; s) \vee \neg d(X_\nu; a(x, s))$
T _{6b}	X	X	X	-	$d(X_\nu; s) \vee d(X_\nu; a(x, s))$
T _{6c}	-	X	X	-	$\neg(d(X_\nu; s) \leftrightarrow d(X_\nu; a(x, s)))$
T ₇	X	X	-	X	$\neg d(X_\nu; s) \vee d(X_\nu; a(x, s))$

TABLE V
TABLE 2 IN [15]

I	II	III	IV	$C_k^a d(X_\nu, x_{\nu+1}; x, s)$
-	-	-	-	-
-	-	X	-	$d(X_\nu; s) \wedge \neg d(X_\nu; a(x_{\nu+1}, s))$
-	-	-	X	$\neg d(X_\nu; s) \wedge \neg d(X_\nu; a(x_{\nu+1}, s))$
-	X	-	-	$\neg d(X_\nu; s) \wedge d(X_\nu; a(x_{\nu+1}, s))$
X	-	-	-	$d(X_\nu; s) \wedge d(X_\nu; a(x_{\nu+1}, s))$
-	-	X	X	$\neg d(X_\nu; a(x_{\nu+1}, s))$
-	X	X	-	$\neg(d(X_\nu; s) \leftrightarrow d(X_\nu; a(x_{\nu+1}, s)))$
X	-	-	X	$d(X_\nu; s) \leftrightarrow d(X_\nu; a(x_{\nu+1}, s))$
X	X	-	-	$d(X_\nu; a(x_{\nu+1}, s))$

that not all rows in Table IV correspond to meaningful norms. (Regarding the distinction between agent-specific norms and system norms, see Sect. 2.1 in [17] with reference to Sergot.) It is argued in [15] that only nine of the 16 rows of Table III are meaningful as the basis for agent-specific norms. They are shown in Table 2 in [15], reiterated here as Table V. On the basis of Table V, we can state prohibitions for an agent $x_{\nu+1}$ in the following way:

Prohibited_{x,s}(x_{ν+1}, a) according to \mathcal{N}
if there exists a condition d
and a condition c and a $k \in$
 $\{\{\text{III}\}, \{\text{IV}\}, \{\text{II}\}, \{\text{I}\}, \{\text{III, IV}\}, \{\text{II, III}\}, \{\text{I, IV}\}, \{\text{I, II}\}\}$
such that $\langle d, P_k c \rangle$ is a norm in \mathcal{N} ,
and there exist x_1, \dots, x_ν such that
 $d(x_1, \dots, x_p, x_{\nu+1}; x, s) \ \& \ C_k^a c(x_1, \dots, x_q, x_{\nu+1}; x, s)$,
where $\nu = \max(p, q)$.

Under the interpretation of E_x/Do that yields Table IV, neither T₅ ('O $E_x F$ ') nor T₇ ('O $E_x \neg F$ ') seem to be useful when formulating agent-specific norms for norm-regulated transition system situations; cf. the discussion in [17, p. 17f]. With another understanding of Do that is consistent with Lindahl's

example in [7, p. 69f], a reasonable interpretation of, e.g., T₅ ('Shall Do(x, F)') as an agent-specific norm is that the agent may act in such a way that F remains true or becomes true, but not in such a way that $\neg F$ remains or becomes true. In other words, III and IV are to be prohibited while I and II are not. However, in the interpretation represented by Table 3, the prohibition of III and IV corresponds to T_{2c}, not to T₅.

So, the question remains whether or not it is possible to find a mapping between the set of nine transition type prohibition operators from Sect. I-C and Lindahl's set of seven types of one-agent normative positions, such that it is suitable as the basis for formulating agent-specific norms and consistent with Lindahl's example. One attempt in this direction follows from an observation in [18], where Odelstad defines three operators Do, Pass and Act in terms of state transition types. A natural understanding of the statement ' x does not see to it that d and does not see to it that not d ' is that it expresses x 's passivity with regard to a state of affairs d , in the sense that the presence or absence of the agent does not affect the truth of d . In the norm-regulated transition system situation context, this corresponds to a behaviour such that x leaves d as it is, no matter if d is true or false; in other words to a behaviour characterised by the transition types I and IV. But, as pointed out by Odelstad, there is another possible understanding of this statement, namely a 'stubbornly active' ('opposive') behaviour such that x changes the truth of d , no matter if d is true or false; i.e. a behaviour characterised by the transition types II and III:

The consistent pairs of (I) – (IV) correspond to Do($\omega, d; s$), Do($\omega, \neg d; s$), Pass($\omega, d; s$) or Act($\omega, d; s$). From this follows that not Do($\omega, d; s$) and not Do($\omega, \neg d; s$) implies Pass($\omega, d; s$) or Act($\omega, d; s$). In [5, p. 147] it is said that x 's passivity with regard to q is expressed by the formula

$$\neg \text{Do}(x, q) \wedge \neg \text{Do}(x, \neg q)$$

TABLE VI
TRANSITION TYPE CONDITIONS FOR EXTENDED TYPES (Λ MNEMONIC FOR LEAVE AND Ω FOR OPPOSE)

Transition type prohibition condition	Corresponding C-operator	$Prohibited_a(x, s)$ if
$P_1^a d$: May Do(x, d) \wedge May Leave(x, d) \wedge May Oppose(x, d) \wedge May Do(x, d')	-	-
$P_{2\Lambda}^a d$: May Do(x, d) \wedge May Leave(x, d) \wedge \neg May Oppose(x, d) \wedge \neg May Do(x, d')	$C_{2\Lambda}^a$	$d(X_\nu; s) \wedge \neg d(X_\nu; a(x, s))$
$P_{2\Omega}^a d$: May Do(x, d) \wedge \neg May Leave(x, d) \wedge May Oppose(x, d) \wedge \neg May Do(x, d')	$C_{2\Omega}^a$	$\neg d(X_\nu; s) \wedge \neg d(X_\nu; a(x, s))$
$P_{4\Lambda}^a d$: \neg May Do(x, d) \wedge May Leave(x, d) \wedge \neg May Oppose(x, d) \wedge May Do(x, d')	$C_{4\Lambda}^a$	$\neg d(X_\nu; s) \wedge d(X_\nu; a(x, s))$
$P_{4\Omega}^a d$: \neg May Do(x, d) \wedge \neg May Leave(x, d) \wedge May Oppose(x, d) \wedge May Do(x, d')	$C_{4\Omega}^a$	$d(X_\nu; s) \wedge d(X_\nu; a(x, s))$
$P_5^a d$: Shall Do(x, d)	C_5^a	$\neg d(X_\nu; a(x, s))$
$P_{6\Lambda}^a d$: Shall Leave(x, d)	$C_{6\Lambda}^a$	$\neg(d(X_\nu; s) \leftrightarrow d(X_\nu; a(x, s)))$
$P_{6\Omega}^a d$: Shall Oppose(x, d)	$C_{6\Omega}^a$	$d(X_\nu; s) \leftrightarrow d(X_\nu; a(x, s))$
$P_7^a d$: Shall Do(x, d')	C_7^a	$d(X_\nu; a(x, s))$

and this is abbreviated as $Pass(x, q)$. But this seems to disregard the possibility that x is with regard to q always active. [18, pp. 42f]

Clearly, ‘opposive’ behaviour is not the same as ‘passive’ behaviour with regard to d , as we intuitively understand ‘passivity’. Neither is it the same as ‘seeing to it that d ’ or to ‘seeing to it that not d ’. Therefore, it is argued that ‘oppositivity’ is another possible reading of $\neg Do(x, d) \wedge \neg Do(x, \neg d)$. (See [17, p. 21f] for an illustrating and motivating example.) This is the idea that Odelstad’s definitions of Do, Pass and Act (in [17] referred to as Do, Leave and Oppose) try to capture.

An analysis based on these ideas was performed in [17, pp. 20ff]. The details are omitted here, but the conclusion is shown in Table VI. A comparison with Table V reveals that there is a natural understanding of the above extension of the Kanger-Lindahl theory of types of one-agent normative positions in terms of the set of nine transition type prohibition operators. That is, in the context of norm-regulated transition system situations, the Kanger-Lindahl type T_1 is represented by the transition type prohibition operator P_1 . Similarly, P_5 represents T_5 and P_7 represents T_7 . T_2 may be represented either by $P_{2\Lambda}$ or $P_{2\Omega}$, and analogously for T_4 and T_6 . It is interesting to note that type T_3 , which is given special attention by Lindahl when developing the theory, has no representation in the context of norm-regulated transition system situations; the suggested interpretation does not allow for the fine-grained distinction between T_1 and T_3 . For example, if we represent $T_6 d$ by the extended type $P_{6\Lambda} d$, which represents the statement Shall Leave(x, d), then act a is prohibited for $x_{\nu+1}$ if $C_{6\Lambda}^a d(x_1, \dots, x_\nu, x_{\nu+1}; x, s)$ for some agents x_1, \dots, x_ν . It is an open problem to state the specific assumptions regarding the logic of Do, Leave, Oppose and Shall that correspond to this interpretation.

2) *Ownership to an estate (continued)*: We are now in the position to formulate a normative system of the Estates world in Sect. I-C1 in terms of prohibition of transition types. Adopting an ‘agent-specific norms’ perspective, the Do-Leave-Oppose interpretation is chosen. For two of the types, T_4 and T_6 , we have the choice of representing these types with $P_{4\Lambda}$ or $P_{4\Omega}$ (resp., $P_{6\Lambda}$ or $P_{6\Omega}$). It is quite clear that, in this example, ‘passivity’ with regard to a condition d is intended to be read as ‘leaving things as they are’ as regards d . Hence, we choose

$P_{4\Lambda}$ and $P_{6\Lambda}$, not $P_{4\Omega}$ and $P_{6\Omega}$. For the grounds, we use the operator M_1 (see [15] for an explanation) to identify the acting agent with the argument to the unary conditions o_1 and o_2 and their boolean combinations. Thus, the following normative system is obtained:

$$\begin{aligned} &\langle M_1(o_1 \wedge o_2), P_1 w_1 \wedge P_1 w_2 \wedge P_1 f_{1,2} \rangle \\ &\langle M_1(o_1 \wedge o'_2), P_1 w_1 \wedge P_{6\Lambda} w_2 \wedge P_{4\Lambda} f_{1,2} \rangle \\ &\langle M_1(o'_1 \wedge o_2), P_6 w_1 \wedge P_1 w_2 \wedge P_{4\Lambda} f_{1,2} \rangle \\ &\langle M_1(o'_1 \wedge o'_2), P_6 w_1 \wedge P_{6\Lambda} w_2 \wedge P_{6\Lambda} f_{1,2} \rangle \end{aligned}$$

Let us recall the specific situation S , in which $s = \{ \langle 1, Alice, W, No \rangle, \langle 2, Bob, B, No \rangle \}$, from Sect. I-C1. What restrictions on Alice’s choice of action does this normative system give? The only ground condition that is fulfilled in this situation is $M_1(o_1 \wedge o'_2)$, since for $x = Alice$, it holds that $o_1(x) \wedge \neg o_2(x)$ and x is the acting agent. Hence, the normative condition that is ‘in effect’ is $P_1 w_1 \wedge P_{6\Lambda} w_2 \wedge P_{4\Lambda} f_{1,2}$. As can be seen in Table VI, P_1 is the most permissible type; $P_1 w_1$ gives no restriction on Alice’s actions. What about $P_{6\Lambda} w_2$, which represents Shall Leave(x, w_2), i.e., that matters should be left as they are regarding the colour of the main building on Blackacre? If $P_{6\Lambda} w_2(Alice, s)$, then if $C_{6\Lambda}^a w_2(Alice, s)$ for some action a , then a is prohibited. But none of the four available actions (i.e., n , $p_{1,W}$, $p_{1,b}$, and $e_{1,2}$) can change the colour of Blackacre’s main building, so for each a it is not the case that $w_2(s) \wedge \neg w_2(s^+)$ or $\neg w_2(s) \wedge w_2(s^+)$, where $s^+ = a(Alice, s)$. Hence, $C_{6\Lambda}^a w_2(Alice, s)$ holds for no $a \in A$, which means that $P_{6\Lambda} w_2$ does not prohibit any of the acts in A . Finally, $P_{4\Lambda} f_{1,2}$, which represents \neg May Do($x, f_{1,2}$) \wedge May Leave($x, f_{1,2}$) \wedge \neg May Oppose($x, f_{1,2}$) \wedge May Do($x, f'_{1,2}$), prohibits an action a if $C_{4\Lambda}^a f_{1,2}(Alice, s)$. Now, $C_{4\Lambda}^a f_{1,2}(Alice, s)$ holds for one action, viz. $e_{1,2}$, since $\neg f_{1,2}(s) \wedge f_{1,2}(s^+)$, where $s^+ = e_{1,2}(Alice, s)$. Hence it follows that $e_{1,2}$ is prohibited for Alice in the situation $\langle Alice, s \rangle$. To summarize, in the given situation the normative system prohibits one of the four available acts in a way that seems to be in line with the intentions in [9, Sect. 4.4.1].

III. APPLICATIONS

An instrumentalisation of the *Estates* example in Sect. I-C1 and II-2 has been developed, using the general-level

Java/Prolog-framework from [11] with later extensions.¹³ By varying the different parameters, one can use the application to experiment with a large number of specific transition system situations, to get an idea of how well the instrumentalised normative system captures the intention behind the system described by Lindahl and Odelstad in [9]. Furthermore, the *Estates* application demonstrates how the framework handles non-elementary norms, i.e., norms whose grounds and/or consequences are boolean combinations of simpler conditions.

IV. CONCLUSION AND FUTURE WORK

An investigation of how to apply the theory of one-agent normative positions in the context of a basic class of transition systems, in which transitions are deterministic and associated with a single agent performing an act, was performed. By interpreting two different extended systems of one-agent types of normative positions in terms of permitting or prohibiting different transition types, two lexicons were obtained for these extended systems in the context of the selected basic class of transition systems. It was demonstrated that both interpretations are natural and useful, depending on how the notion of agency (in the theory of normative positions represented by the action operator Do) is understood and whether a ‘system norms’ or an ‘agent-specific norms’ perspective is taken.

A contribution to a typology of interpretations of the theory of normative positions was thus made, with future work including to explicitly state additional assumptions regarding the logic of Do, Leave, Oppose and May that capture the meaning of these operators in a norm-regulated transition system situation context, and further investigating the formal algebraic aspects of representing normative systems under these interpretations. This includes formulating the equivalent of a *normative position condition-implication structure* (abbreviated *np-cis*; cf. for example [5, p. 157]) for the extended systems of normative positions discussed here. It would also be interesting to generalise the concept of transition system situations, and study how Lindahl’s system of two-agent types of normative positions could be used to deal with simultaneous actions by two agents (including ‘actions’ by the environment) in this context. Finally, an analysis of experiments with the *Estates* application might shed some more light on the benefits and limits of the approach to normative systems presented here.

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¹³The application is not presented in detail here, due to lack of space. The source code is publicly available for download from <http://drp.name/norms/nrtssit/>