Normative Positions within Norm-regulated Transition System Situations

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Abstract. It is possible to find a natural understanding of the Kanger-Lindahl theory of normative positions in the context of *norm-regulated transition system situations*, in which the permission or prohibition of actions is related to the permission or prohibition of different types of state transitions with respect to some condition on agents. Lexicons for two systems of types of normative positions are suggested and discussed. It is demonstrated that both interpretations are natural and useful, depending on how the notion of agency is understood and whether a 'system norms' or an 'agent-specific norms' perspective is taken.
Keywords: transition system, multi-agent system, norm-regulated, norm-governed, normative system, normative positions, deontic positions, deontic logic

1 Introduction

The study of norm-regulated multi-agent systems covers areas such as the formal representation and implementation of normative systems as well as applications of norm-regulated MAS. One approach to the design of normative systems and the formulation of norms is the use of if-then-else rules or constraints on the states, and the transitions between states, of an agent or of the system as a whole. In many systems, the actions of an individual agent are naturally associated with transitions between different states of the system. As a consequence, the permission or prohibition of a specific action in such a system is connected to permissible or prohibited transitions between states of the system, and norms may then be formulated as restrictions on states and state transitions. The 'agent-stranded transition systems' framework by Craven and Sergot [1, 19] and the Ballroom system by Gaertner et al. [2] both serve as examples of this approach. Other approaches are algebraic or based on modal logics, like temporal or deontic logic. One example of the latter is the combination by Governatori et al. [3] of temporalised agency and temporalised normative positions, in the setting of Defeasible Logic. The DALMAS architecture for norm-regulated MAS [18] is based on an algebraic approach to the representation of normative systems. The present author has introduced the notion of *norm-regulated transition system situations* in an attempt to show that the basic ideas underlying the DALMAS architecture are applicable to a larger class of MAS. In this approach, the permission or prohibition of actions is related to the permission or prohibition of different types of state transitions with respect to some condition d on a number of agents $x_1, ..., x_n$ in a state.

The Kanger-Lindahl system of one-agent types of normative positions[3] and various extensions are presented. From a theoretical point of view, the weak underlying assumptions makes the theory of normative positions elegant and general. On the other hand, this allows for great freedom when creating models for the theory, which becomes a challenge for practical applications. We investigate whether or not it is possible to find a natural understanding of the theory of normative positions in the context of norm-regulated transition system situations. As a first step towards a typology of models for the theory, different lexicons are suggested and discussed.[4]

1.1 One-Agent Types of Normative Positions

The Kanger-Lindahl theory of normative positions is based on Kanger's 'deontic action-logic' [11, 12]. The theory, further developed by Lindahl in [13], contains three systems of types of normative positions. The simplest of these systems is a system of seven 'one-agent types' of normative positions, based on the logic

[3] The term 'deontic positions' has been suggested as a replacement for 'normative positions', but the latter term will be used here.

[4] Throughout the text, terms such as 'interpretation', 'understanding' and 'lexicon' are used in an informal sense and more or less interchangeably.

of the action operator Do and the deontic operator Shall. $\mathrm{Do}(x, d)$ is read as 'x sees to it that d' or 'x brings it about that d'. The logical properties assumed for Do is that it is the smallest system containing propositional logic, closed under logical equivalence and containing the axiom schema $\mathrm{Do}(x, d) \rightarrow d$. The latter schema tries to capture the notion of *successful* action; if x 'sees to it' or 'brings about' that d, then d is indeed the case.

Each of the three statements (i) $\mathrm{Do}(x, d)$, (ii) $\mathrm{Do}(x, \neg d)$ and (iii) $\neg\mathrm{Do}(x, d) \wedge \neg\mathrm{Do}(x, \neg d)$ implies the negation of each of the others, and the disjunction of all three is a tautology. Each of (i) - (iii) can be prefixed with either May or ¬May, where May F is defined as $\neg\mathrm{Shall}\neg F$, and basic conjunctions containing one statement from each such pair are formed. By iterated construction of basic conjunctions, a set of eight conjunctions is obtained. One such 'maxi-conjunction' is self-contradictory, the other seven are listed below:

- $\mathrm{T}_1(x, d) : \mathrm{May\,Do}(x, d) \wedge \mathrm{May}[\neg\mathrm{Do}(x, d) \wedge \neg\mathrm{Do}(x, \neg d)] \wedge \mathrm{May\,Do}(x, \neg d)$;
- $\mathrm{T}_2(x, d) : \mathrm{May\,Do}(x, d) \wedge \mathrm{May}[\neg\mathrm{Do}(x, d) \wedge \neg\mathrm{Do}(x, \neg d)] \wedge \neg\mathrm{May\,Do}(x, \neg d)$;
- $\mathrm{T}_3(x, d) : \mathrm{May\,Do}(x, d) \wedge \neg\mathrm{May}[\neg\mathrm{Do}(x, d) \wedge \neg\mathrm{Do}(x, \neg d)] \wedge \mathrm{May\,Do}(x, \neg d)$;
- $\mathrm{T}_4(x, d) : \neg\mathrm{May\,Do}(x, d) \wedge \mathrm{May}[\neg\mathrm{Do}(x, d) \wedge \neg\mathrm{Do}(x, \neg d)] \wedge \mathrm{May\,Do}(x, \neg d)$;
- $\mathrm{T}_5(x, d) : \mathrm{May\,Do}(x, d) \wedge \neg\mathrm{May}[\neg\mathrm{Do}(x, d) \wedge \neg\mathrm{Do}(x, \neg d)] \wedge \neg\mathrm{May\,Do}(x, \neg d)$;
- $\mathrm{T}_6(x, d) : \neg\mathrm{May\,Do}(x, d) \wedge \mathrm{May}[\neg\mathrm{Do}(x, d) \wedge \neg\mathrm{Do}(x, \neg d)] \wedge \neg\mathrm{May\,Do}(x, \neg d)$;
- $\mathrm{T}_7(x, d) : \neg\mathrm{May\,Do}(x, d) \wedge \neg\mathrm{May}[\neg\mathrm{Do}(x, d) \wedge \neg\mathrm{Do}(x, \neg d)] \wedge \mathrm{May\,Do}(x, \neg d)$.

Some further extensions of the systems of normative positions has been suggested by Jones and Sergot [9, 20]; cf. Sect. 1.2. They have explored some applications of the theory within computer science, and discussed some of its limitations in this setting. Lindahl and Odelstad [14, 15] have combined the theory of normative positions with an algebraic approach to normative systems. DALMAS [18] is an abstract architecture for a class of multi-agent systems, regulated by normative systems based on this approach. A general-level Java/Prolog implementation of the DALMAS architecture has been developed [4, 8], to facilitate the implementation of specific systems.

1.2 An Extended Set of One-Agent Types of Normative Positions

Jones and Sergot [9, pp. 18-19] have generalised and further developed the Kanger-Lindahl theory of normative position. Using a method suitable for automation, they perform a similar analysis as Lindahl. First, a set of 'act positions'

1. $\mathrm{E}_x F$
2. $\mathrm{E}_x \neg F$
3. $\neg\mathrm{E}_x F \wedge \neg\mathrm{E}_x \neg F$

is generated from the scheme $\|\pm\mathrm{E}_x \pm F\|$.[5] Using O for Shall and P for May, each of the three logically consistent act positions is then prefixed with ±O±, which

[5] The operator E_x corresponds to the Kanger-Lindahl Do operator. $\|\pm\mathrm{E}_x \pm F\|$ stands for the set of *maxi-conjunctions* of $\pm\mathrm{E}_x \pm F$, i.e., the maximal consistent conjunctions of expressions of the form $\pm\mathrm{E}_x \pm F$, where ± stands for the two possibilites of affirmation and negation. This notation was suggested by Makinson [16].

Table 1. Table 2 in [20]

$$\mathrm{T}_1 \left\{ \begin{array}{c} \mathrm{PE}_a F \wedge \mathrm{PE}_a \neg F \wedge \mathrm{P}(F \wedge \neg \mathrm{E}_a F) \wedge \mathrm{P}(\neg F \wedge \neg \mathrm{E}_a \neg F) \\ \mathrm{PE}_a F \wedge \mathrm{PE}_a \neg F \wedge \neg \mathrm{P}(F \wedge \neg \mathrm{E}_a F) \wedge \mathrm{P}(\neg F \wedge \neg \mathrm{E}_a \neg F) \\ \mathrm{PE}_a F \wedge \mathrm{PE}_a \neg F \wedge \mathrm{P}(F \wedge \neg \mathrm{E}_a F) \wedge \neg \mathrm{P}(\neg F \wedge \neg \mathrm{E}_a \neg F) \end{array} \right\}$$

$$\mathrm{T}_2 \left\{ \begin{array}{c} \mathrm{PE}_a F \wedge \neg \mathrm{PE}_a \neg F \wedge \mathrm{P}(F \wedge \neg \mathrm{E}_a F) \wedge \mathrm{P}(\neg F \wedge \neg \mathrm{E}_a \neg F) \\ \mathrm{PE}_a F \wedge \neg \mathrm{PE}_a \neg F \wedge \neg \mathrm{P}(F \wedge \neg \mathrm{E}_a F) \wedge \mathrm{P}(\neg F \wedge \neg \mathrm{E}_a \neg F) \\ \mathrm{PE}_a F \wedge \neg \mathrm{PE}_a \neg F \wedge \mathrm{P}(F \wedge \neg \mathrm{E}_a F) \wedge \neg \mathrm{P}(\neg F \wedge \neg \mathrm{E}_a \neg F) \end{array} \right\}$$

$$\mathrm{T}_3 \ \{\mathrm{PE}_a F \wedge \mathrm{PE}_a \neg F \wedge \neg \mathrm{PPass}_a F\}$$

$$\mathrm{T}_4 \left\{ \begin{array}{c} \neg \mathrm{PE}_a F \wedge \mathrm{PE}_a \neg F \wedge \mathrm{P}(F \wedge \neg \mathrm{E}_a F) \wedge \mathrm{P}(\neg F \wedge \neg \mathrm{E}_a \neg F) \\ \neg \mathrm{PE}_a F \wedge \mathrm{PE}_a \neg F \wedge \neg \mathrm{P}(F \wedge \neg \mathrm{E}_a F) \wedge \mathrm{P}(\neg F \wedge \neg \mathrm{E}_a \neg F) \\ \neg \mathrm{PE}_a F \wedge \mathrm{PE}_a \neg F \wedge \mathrm{P}(F \wedge \neg \mathrm{E}_a F) \wedge \neg \mathrm{P}(\neg F \wedge \neg \mathrm{E}_a \neg F) \end{array} \right\}$$

$$\mathrm{T}_5 \ \{\mathrm{OE}_a F\}$$

$$\mathrm{T}_6 \left\{ \begin{array}{c} \mathrm{OPass}_a F \wedge \mathrm{O} F \\ \mathrm{OPass}_a F \wedge \mathrm{O} \neg F \\ \mathrm{OPass}_a F \wedge \mathrm{P} F \wedge \mathrm{P} \neg F \end{array} \right\}$$

$$T_7 \ \{\mathrm{OE}_a \neg F\}$$

yields a set of 64 maxi-conjunctions. Assuming the same logical properties of the O and E_x operators as in Lindahl's analysis, viz., the axiom of successful action, E.T., and closure under logical equivalence, E.RE., 57 of the 64 conjunctions are internally inconsistent. The remaining seven are precisely Lindahl's seven one-agent types of normative positions:

$$\|\pm\mathrm{O} \pm \|\pm\mathrm{E}_x \pm F\|\| = \|\pm\mathrm{P}\ \|\pm\mathrm{E}_x \pm F\|\| \tag{1}$$

For weaker logics this equality does not hold. The consequences of alternative logics of O or E_x will not be explored here. Instead, let us consider Sergot's idea to perform a refined analysis, based on the set of four 'cumulative fact/act positions'

1. (A_1) $\mathrm{E}_x F$
2. (A_2) $\mathrm{E}_x \neg F$
3. (A_{3a}) $F \wedge \neg \mathrm{E}_x F$
4. (A_{3b}) $\neg F \wedge \neg \mathrm{E}_x \neg F$

which is generated from the scheme $\|\pm\mathrm{E}_x \pm F\| \cdot \|\pm F\|$. As before, these conjunctions are then prefixed by $\pm\mathrm{O}\pm$:

$$\|\pm\mathrm{O} \pm \|\pm\mathrm{E}_x \pm F\| \cdot \|\pm F\|\| \tag{2}$$

This analysis yields a set of 15 logically consistent conjunctions, shown in Table 2 in [20, p. 375]. (The table is reiterated here as Table 1. Note that $\mathrm{Pass}_a F$ is an abbreviation for $\neg \mathrm{E}_a F \wedge \neg \mathrm{E}_a \neg F$.)

Lindahl's T_3, T_5 and T_7 are identical to three of Sergot's 'normative act positions', while each of the other four types are logically equivalent to a disjunction of three conjunctions, as shown in the table.

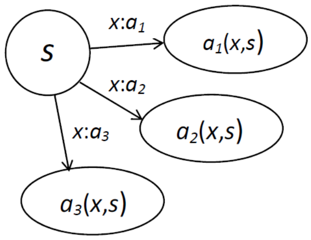

Fig. 1. A state diagram for a transition system situation with three events.

1.3 Previous Work: Norm-regulated Transition System Situations

The notion of a norm-regulated transition system situation was originally presented in [6]. The normative framework uses an algebraic representation of conditional norms and is based on a systematic exploration of the possible types of state transitions with respect to some state of affairs $d(x_1, ..., x_n)$.

A *transition system situation* is an ordered 5-tuple $\mathbf{S} = \langle x, s, A, \Omega, S \rangle$ characterised by a set of states S, a state s, an agent-set $\Omega = \{x_1, ..., x_n\}$, the acting ('moving') agent x, and an action-set $A = \{a_1, ..., a_m\}$. In this setting, a may be regarded as a function such that $a(x, s) = s^+$ means that s^+ is the resulting state when x performs act a in state s. In the following, the abbreviation s^+ will be used for $a(x, s)$ when there is no need for an explicit reference to the action a and the acting agent x. It is assumed that the action by the acting agent is deterministic and is performed asynchronously, i.e., that there is no simultaneous action by other agents (including the 'environment', which may be regarded as a special kind of agent). Furthermore, we assume that a ν-ary condition d is true or false on ν agents $x_1, ..., x_\nu \in \Omega$ in s, where Ω is a set of agents associated with s; this will be written $d(x_1, ..., x_\nu; s)$. To facilitate the presentation, X_ν will often be used as an abbreviation for the argument sequence $x_1, ..., x_\nu$.[6]

A transition system situation is intended to represent, for example, a 'snapshot' of a labelled transition system (LTS) in which each transition is deterministic and represents the action of a single agent. In this case, s represents an arbitrary state in the LTS, and S is the set of states reachable from s by all transitions $x{:}a$, $a \in A$. See Fig. 1 for an example.

Now consider the transition from a state s to a following state s^+, and focus on the state of affairs $d(X_\nu)$. With regard to $d(X_\nu)$, there are four possible alternatives for the transition from s to s^+, since in s as well as in s^+, $d(X_\nu)$ or $\neg d(X_\nu)$ could hold[7]:

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[6] In the special case when the sequence of agents is empty, i.e. $\nu = 0$, d represents a proposition which is true or false in s.

[7] Negations d' of conditions can be formed in the following way: $d'(X_\nu)$ iff $\neg d(X_\nu)$. In the following, the latter notation will be used to facilitate the presentation. Con-

I. $d(X_\nu; s)$ and $d(X_\nu; s^+)$

II. $\neg d(X_\nu; s)$ and $d(X_\nu; s^+)$

III. $d(X_\nu; s)$ and $\neg d(X_\nu; s^+)$

IV. $\neg d(X_\nu; s)$ and $\neg d(X_\nu; s^+)$

Each alternative represents a basic type of transition with regard to the state of affairs $d(X_\nu)$; we say that $\{\mathrm{I}, \mathrm{II}, \mathrm{III}, \mathrm{IV}\}$ is the set of *basic transition types* with regard to $d(X_\nu)$. The assignment of different types (with regard to some state of affairs) to state transitions closely resembles Sergot's ideas in [19].[8] The main contribution here is that the types are systematically explored, and that the state of affairs is a condition which is true or false of a number of agents, not merely a proposition. Another notable difference is that, in Sergot's framework, the agent is said to bring it about that a transition has a certain type, rather than bringing about a certain state of affairs.

Let the *situation* $\langle x, s\rangle$ be characterised by the moving agent x and the state s in a transition system situation $\mathbf{S}$. To be able to determine the type, with respect to $d(X_\nu)$, of the transition represented by action a performed by agent $x_{\nu+1}$ in $\langle x, s\rangle$[9], we define a 'basic transition type operator' B^a_j, $j \in \{\mathrm{I}, \mathrm{II}, \mathrm{III}, \mathrm{IV}\}$, such that the $\nu+1$-ary 'transition type condition' $B^a_j d(X_\nu, x_{\nu+1}; x, s)$ indicates whether or not, in the situation $\langle x, s\rangle$, the event $x_{\nu+1}{:}a$ (representing a being performed by $x_{\nu+1}$) has basic transition type j with regard to $d(X_\nu)$: For all ν-ary conditions d and for all agents $X_\nu, x_{\nu+1}$, all acts a and all situations $\langle x, s\rangle$,

I. $B^a_{\mathrm{I}} d(X_\nu, x_{\nu+1}; x, s)$ iff $[d(X_\nu; s) \wedge d(X_\nu; a(x_{\nu+1}, s)]$

II. $B^a_{\mathrm{II}} d(X_\nu, x_{\nu+1}; x, s)$ iff $[\neg d(X_\nu; s) \wedge d(X_\nu; a(x_{\nu+1}, s))]$

III. $B^a_{\mathrm{III}} d(X_\nu, x_{\nu+1}; x, s)$ iff $[d(X_\nu; s) \wedge \neg d(X_\nu; a(x_{\nu+1}, s))]$

IV. $B^a_{\mathrm{IV}} d(X_\nu, x_{\nu+1}; x, s)$ iff $[\neg d(X_\nu; s) \wedge \neg d(X_\nu; a(x_{\nu+1}, s))]$

A *norm-regulated transition system situation* is represented by an ordered pair $\langle \mathbf{S}, \mathcal{N}\rangle$ where $\mathbf{S} = \langle x, s, A, \Omega, S\rangle$ is a transition system situation and $\mathcal{N}$ is a normative system. It is assumed that norms apply to an individual agent $x_{\nu+1}$ in a state s. A norm in $\mathcal{N}$ is represented by an ordered pair $\langle G, C\rangle$, where the (descriptive) condition G on a situation $\langle x, s\rangle$ is the *ground* of the norm and the (normative) condition C on $\langle x, s\rangle$ is its *consequence*. (See, e.g., [18].) Let us define a set of 'transition type prohibition operators' P_k, $k \in \mathcal{P}(\{\mathrm{I}, \mathrm{II}, \mathrm{III}, \mathrm{IV}\})$, where $\mathcal{P}(S)$ is the power set of S, such that $P_k d(X_\nu, x_{\nu+1}; x, s)$ indicates that the basic transition types (with respect to the state of affairs $d(X_\nu)$) in k are prohibited, and a set of corresponding 'transition type operators' C^a_k, such that

junctions ($c \wedge d$) and disjunctions ($c \vee d$) may be formed in a similar way; hence, it is possible to construct Boolean algebras of conditions.

[8] In the vein of [19], I could be written $0{:}d(X_\nu) \wedge 1{:}d(X_\nu)$, II could be written $0{:}\neg d(X_\nu) \wedge 1{:}d(X_\nu)$, and similarly for III and IV.

[9] Note that, in most systems, $x_{\nu+1} = x$. See [6] for an explanation.

$C_k^a d(X_\nu, x_{\nu+1}; x, s)$ iff the transition from s to $a(x, s)$ has any of the basic transition types in k with respect to $d(X_\nu)$.[10] Then, for example, $\langle c, P_k d\rangle$ represents the sentence

$$\forall x_1, x_2, ..., x_\nu, x_{\nu+1} \in \Omega : c(x_1, x_2, ..., x_p, x_{\nu+1}; x, s) \rightarrow \\ P_k d(x_1, x_2, ..., x_q, x_{\nu+1}; x, s)$$

where Ω is the set of agents, $x_{\nu+1}$ is the agent to which the norm applies, x is the acting agent in the situation $\langle x, s\rangle$, and $\nu = \max(p, q)$. If the condition specified by the ground of a norm is true in some situation, then the (normative) consequence of the norm is in effect in that situation. If the normative system contains a norm whose ground holds in the situation $\langle x, s\rangle$ and whose consequence prohibits the type of transition represented by the event $x_{\nu+1}{:}a$, then action a is prohibited for $x_{\nu+1}$ in $\langle x, s\rangle$:

$$\begin{gathered} \mathit{Prohibited}_{x,s}(x_{\nu+1}, a) \text{ according to } \mathcal{N} \\ \text{if there exists a condition } c \text{ and a condition } d \text{ and a } k \in \mathcal{P}(\{\mathrm{I}, \mathrm{II}, \mathrm{III}, \mathrm{IV}\}), \\ \text{such that } \langle c, P_k d\rangle \text{ is a norm in } \mathcal{N}, \text{ and there exist } x_1, ..., x_\nu \text{ such that} \\ c(x_1, ..., x_p, x_{\nu+1}; x, s) \;\&\; C_k^a d(x_1, ..., x_q, x_{\nu+1}; x, s), \text{ where } \nu = \max(p, q). \end{gathered}$$

Hence, if $P_k d$ is 'in effect' and $C_k^a d(x_1, ..., x_q, x_{\nu+1}; x, s)$ holds for some sequence of agents $x_1, ..., x_q, x_{\nu+1}$, then a is prohibited for $x_{\nu+1}$ in s. For example,

$$\begin{gathered} \text{if } P_{\{\mathrm{I},\mathrm{IV}\}} d, \text{ then } a \text{ is prohibited for } x_{\nu+1} \text{ in } s \\ \text{if } C^a_{\{\mathrm{I},\mathrm{IV}\}} d(x_1, ..., x_q, x_{\nu+1}; x, s) \\ \text{iff } B^a_{\mathrm{I}} d(X_\nu, x_{\nu+1}; x, s) \text{ or } B^a_{\mathrm{IV}} d(X_\nu, x_{\nu+1}; x, s). \end{gathered}$$

The set of operators C_k^a is derived from the freely generated Boolean algebra over $d(x_1, ..., x_\nu; s)$ and $d(x_1, ..., x_\nu; a(x, s))$; see Table 1 in [6] (reiterated here as Table 2). It is interesting to note that there are 15 conjunctions in Sergot's set of 'normative act positions' (see Sect. 1.2), and also 15 rows in Table 2 (if we disregard the last row that prohibits all state transitions). The following section suggests an interpretation of E_x that lets us identify each of the 'normative act positions' with one of the rows in the table.

2 Mapping Normative Positions to Transition Type Prohibition Operators

Let us try to capture the meaning of Sergot's normative act positions in a transition system situation context. One way of reading $\mathrm{E}_x F$ is 'x brings it about that F', or even 'it is x that brings it about that F'; if F would be (or become) the case anyway, without intervention from the agent x, then it is not the case that

[10] That is, P_k and C_k^a transform ν-ary conditions d on a state s to $\nu+1$-ary conditions $P_k d$ and $C_k^a d$ on a situation $\langle x, s\rangle$.

Table 2. Table 1 in [6]: Possible Combinations of Basic Transition Types.

	I	II	III	IV	
	-	-	-	-	-
$C^a_{\{IV\}}$	-	-	-	X	$\neg d(X_\nu; s) \wedge \neg d(X_\nu; s^+)$
$C^a_{\{III\}}$	-	-	X	-	$d(X_\nu; s) \wedge \neg d(X_\nu; s^+)$
$C^a_{\{III,IV\}}$	-	-	X	X	$\neg d(X_\nu; s^+)$
$C^a_{\{II\}}$	-	X	-	-	$\neg d(X_\nu; s) \wedge d(X_\nu; s^+)$
$C^a_{\{II,IV\}}$	-	X	-	X	$\neg d(X_\nu; s)$
$C^a_{\{II,III\}}$	-	X	X	-	$\neg(d(X_\nu; s) \leftrightarrow d(X_\nu; s^+))$
$C^a_{\{II,III,IV\}}$	-	X	X	X	$\neg d(X_\nu; s) \vee \neg d(X_\nu; a(x, s))$
$C^a_{\{I\}}$	X	-	-	-	$d(X_\nu; s) \wedge d(X_\nu; s^+)$
$C^a_{\{I,IV\}}$	X	-	-	X	$d(X_\nu; s) \leftrightarrow d(X_\nu; s^+)$
$C^a_{\{I,III\}}$	X	-	X	-	$d(X_\nu; s)$
$C^a_{\{I,III,IV\}}$	X	-	X	X	$d(X_\nu; s) \vee \neg d(X_\nu; s^+)$
$C^a_{\{I,II\}}$	X	X	-	-	$d(X_\nu; s^+)$
$C^a_{\{I,II,IV\}}$	X	X	-	X	$\neg d(X_\nu; s) \vee d(X_\nu; s^+)$
$C^a_{\{I,II,III\}}$	X	X	X	-	$d(X_\nu; s) \vee d(X_\nu; s^+)$
$C^a_{\{I,II,II,IV\}}$	X	X	X	X	$\top$

it is x that brings it about that F.[11] Under this interpretation it is reasonable to identify $\mathrm{E}_x F$ with a state transition of type II in a norm-regulated transition system situation.[12] Consequently, a natural understanding of $\neg\mathrm{P}\,\mathrm{E}_x F$ is that II is not allowed. Similar arguments may be given for $\neg\mathrm{P}\,\mathrm{E}_x\neg F$, $\neg\mathrm{P}(\neg\mathrm{E}_x F \wedge F)$ and $\neg\mathrm{P}(\neg\mathrm{E}_x\neg F \wedge \neg F)$. To summarise the discussion in [5, pp. 17f], the following principles can be assumed:

1. $\neg\mathrm{P}\,\mathrm{E}_x F$ implies that II is disallowed.
2. $\neg\mathrm{P}\,\mathrm{E}_x\neg F$ implies that III is disallowed.
3. $\neg\mathrm{P}(\neg\mathrm{E}_x F \wedge F)$ implies that I is disallowed.
4. $\neg\mathrm{P}(\neg\mathrm{E}_x\neg F \wedge \neg F)$ implies that IV is disallowed.

Using these principles, we identify Sergot's 'normative act positions' with a corresponding row in Table 2. The result is shown in Table 3, with the rows ordered as in Sergot's Table II.[13] The table suggests that, following the principles stated above, there is a straightforward interpretation of Sergot's 'normative act positions' in terms of permissible and prohibited state transitions in the context of norm-regulated transition system situations.

Given Sergot's extension of the theory and Table 3, one might say that the problem of finding a natural understanding of the Kanger-Lindahl theory of normative positions in the context of norm-regulated transition system situations

[11] See, e.g., [10]. For a further discussion of different ways of understanding the agency operator Do/E_x, see Sect. 3.1 in [5].

[12] Clearly, this is consistent with the assumption that $\mathrm{E}_x F \to F$.

[13] In order to adhere to the notation in [6], F is represented by $d(x_1, ..., x_\nu)$. With explicit reference to states, we write $d(X_\nu; s)$ or $d(X_\nu; a(x, s))$, and so on.

Table 3. Possible Interpretation of Sergot's Normative Act Positions

	I: $\neg\mathrm{E}_x F \wedge F$	II: $\mathrm{E}_x F$	III: $\mathrm{E}_x \neg F$	IV: $\neg\mathrm{E}_x \neg F \wedge \neg F$	$Prohibited_a(x, s)$ if
T_{1a}	-	-	-	-	-
T_{1b}	X	-	-	-	$d(X_\nu; s) \wedge d(X_\nu; a(x, s))$
T_{1c}	-	-	-	X	$\neg d(X_\nu; s) \wedge \neg d(X_\nu; a(x, s))$
T_{2a}	-	-	X	-	$d(X_\nu; s) \wedge \neg d(X_\nu; a(x, s))$
T_{2b}	X	-	X	-	$d(X_\nu; s)$
T_{2c}	-	-	X	X	$\neg d(X_\nu; a(x, s))$
T_3	X	-	-	X	$d(X_\nu; s) \leftrightarrow d(X_\nu; a(x, s))$
T_{4a}	-	X	-	-	$\neg d(X_\nu; s) \wedge d(X_\nu; a(x, s))$
T_{4b}	X	X	-	-	$d(X_\nu; a(x, s))$
T_{4c}	-	X	-	X	$\neg d(X_\nu; s)$
T_5	X	-	X	X	$d(X_\nu; s) \vee \neg d(X_\nu; a(x, s))$
T_{6a}	-	X	X	X	$\neg d(X_\nu; s) \vee \neg d(X_\nu; a(x, s))$
T_{6b}	X	X	X	-	$d(X_\nu; s) \vee d(X_\nu; a(x, s))$
T_{6c}	-	X	X	-	$\neg(d(X_\nu; s) \leftrightarrow d(X_\nu; a(x, s)))$
T_7	X	X	-	X	$\neg d(X_\nu; s) \vee d(X_\nu; a(x, s))$

is solved. The immediate appeal of the suggested understanding of Sergot's normative act positions is that it is both simple and intuitive. So why not settle for this?

The answer may be a matter of different perspectives of norms and normative systems. If, for example, we are interested in formulating *system norms*, i.e., norms which "... express a system designer's point of view of what system states and transitions are legal, permitted, desirable, and so on" [19, p. 2], then the suggested understanding of normative act positions within the context of norm-regulated transition system situations is reasonable. On the other hand, if we talk about *agent-specific norms*, i.e., norms that apply to an individual agent in a specific situation to regulate which actions the agent may or may not perform, it can be argued that not all rows in Table 3 correspond to meaningful norms. (Regarding the distinction between agent-specific norms and system norms, see Sect. 2.1 in [5] with reference to Sergot.) It is argued in [6] that only nine of the 16 rows of Table 2 are meaningful as the basis for agent-specific norms. They are shown in Table 2 in [6], reiterated here as Table 4. On the basis of Table 4, we can state prohibitions for an agent $x_{\nu+1}$ in the following way:

$$\begin{gathered} Prohibited_{x,s}(x_{\nu+1}, a) \text{ according to } \mathcal{N} \\ \text{if there exists a condition } d \text{ and a condition } c \\ \text{and a } k \in \{\{\mathrm{III}\}, \{\mathrm{IV}\}, \{\mathrm{II}\}, \{\mathrm{I}\}, \{\mathrm{III}, \mathrm{IV}\}, \{\mathrm{II}, \mathrm{III}\}, \{\mathrm{I}, \mathrm{IV}\}, \{\mathrm{I}, \mathrm{II}\}\} \\ \text{such that } \langle d, P_k c\rangle \text{ is a norm in } \mathcal{N}\text{, and there exist } x_1, ..., x_\nu \text{ such that} \\ d(x_1, ..., x_p, x_{\nu+1}; x, s) \;\&\; C_k^a c(x_1, ..., x_q, x_{\nu+1}; x, s), \text{ where } \nu = \max(p, q). \end{gathered}$$

Under the interpretation of E_x/Do that yields Table 3, neither T_5 ('$\mathrm{O}\,\mathrm{E}_x F$') nor T_7 ('$\mathrm{O}\,\mathrm{E}_x \neg F$') seem to be useful when formulating agent-specific norms for

Table 4. Table 2 in [6].

I	II	III	IV	$C^a_k d(X_\nu, x_{\nu+1}; x, s)$
-	-	-	-	-
-	-	X	-	$d(X_\nu; s) \wedge \neg d(X_\nu; a(x_{\nu+1}, s))$
-	-	-	X	$\neg d(X_\nu; s) \wedge \neg d(X_\nu; a(x_{\nu+1}, s))$
-	X	-	-	$\neg d(X_\nu; s) \wedge d(X_\nu; a(x_{\nu+1}, s))$
X	-	-	-	$d(X_\nu; s) \wedge d(X_\nu; a(x_{\nu+1}, s))$
-	-	X	X	$\neg d(X_\nu; a(x_{\nu+1}, s))$
-	X	X	-	$\neg(d(X_\nu; s) \leftrightarrow d(X_\nu; a(x_{\nu+1}, s)))$
X	-	-	X	$d(X_\nu; s) \leftrightarrow d(X_\nu; a(x_{\nu+1}, s))$
X	X	-	-	$d(X_\nu; a(x_{\nu+1}, s))$

norm-regulated transition system situations; cf. the discussion in [5, p. 17f]. It is, however, clear from Lindahl's example in [13, p. 69f] that one can have another interpretation in mind, where an agent x can 'see to it that' F either 'actively' or by performing a 'null action' with respect to F, depending on whether or not F is already the case. With this understanding, a reasonable interpretation of, e.g., T_5 ('Shall Do(x, F)') as an agent-specific norm is that the agent may act in such a way that F remains true or becomes true, but not in such a way that $\neg F$ remains or becomes true. In other words, III and IV are prohibited while I and II are not. However, in the interpretation represented by Table 3, the prohibition of III and IV corresponds to T_{2c}, not to T_5.

So, the question remains whether or not it is possible to find a mapping between the set of nine transition type prohibition operators from Sect. 1.3 and Lindahl's set of seven types of one-agent normative positions, such that it is suitable as the basis for formulating agent-specific norms and consistent with Lindahl's example. One attempt in this direction follows from an observation in [17]. Odelstad defines three operators Do, Pass and Act [17, pp. 42f] in terms of state transition types. (The discussion, omitted here due to lack of space, is summarised in [5, pp. 20ff].)

A natural understanding of the statement 'x does not see to it that d and does not see to it that not d' is that it expresses x's *passivity* with regard to a state of affairs d, in the sense that the presence or absence of the agent does not affect the truth of d. In the norm-regulated transition system situation context, this corresponds to a behaviour such that x leaves d as it is, no matter if d is true or false; in other words to a behaviour characterised by the transition types I and IV. But, as pointed out by Odelstad, there is another possible understanding of this statement, namely a 'stubbornly active' ('opposive') behaviour such that x changes the truth of d, no matter if d is true or false; i.e. a behaviour characterised by the transition types II and III:

> The consistent pairs of (I) – (IV) correspond to $\mathrm{Do}(\omega, d; s)$, $\mathrm{Do}(\omega, \neg d; s)$, $\mathrm{Pass}(\omega, d; s)$ or $\mathrm{Act}(\omega, d; s)$. From this follows that not $\mathrm{Do}(\omega, d; s)$ and not $\mathrm{Do}(\omega, \neg d; s)$ implies $\mathrm{Pass}(\omega, d; s)$ or $\mathrm{Act}(\omega, d; s)$. In [18, p. 147] it is said that x's passivity with regard to q is expressed by the formula

> $\neg\mathrm{Do}(x, q) \wedge \neg\mathrm{Do}(x, \neg q)$
>
> and this is abbreviated as $\mathrm{Pass}(x, q)$. But this seems to disregard the possibility that x is with regard to q always active. [17, pp. 42f]

Clearly, 'opposive' behaviour is not the same as 'passive' behaviour with regard to d, as we intuitively understand 'passivity'. Neither is it the same as 'seeing to it that d' or to 'seeing to it that not d'. Therefore, it is argued that 'opposivity' is another possible understanding of $\neg\mathrm{Do}(x, d) \wedge \neg\mathrm{Do}(x, \neg d)$. (See [5, p. 21f] for an illustrating and motivating example.) This is the idea that Odelstad's definitions of Do, Pass and Act (in [5] referred to as Do, Leave and Oppose) try to capture.

An analysis based on the ideas above was performed in [5, pp. 20ff]. The details are left out here, but the conclusion is shown in Table 5. By comparing Table 5 with Table 4, we can see that there is a natural understanding of the above extension of the Kanger-Lindahl theory of types of one-agent normative positions in terms of the set of nine transition type prohibition operators. For example, the extended type $P_{6\Lambda}d$, which represents the statement Shall Leave(x, d), prohibits act a for $x_{\nu+1}$ if $C^a_{6\Lambda}d(x_1, ..., x_\nu, x_{\nu+1}; x, s)$ for some agents $x_1, ..., x_\nu$. It is an open problem to state the specific additional assumptions regarding the logic of Do, Leave, Oppose and May that correspond to this interpretation.

3 Applications

A number of running example systems based on norm-regulated transition system situations (see [6, Sect. 2.3] for an overview) have been built, using the general-level Java/Prolog-framework from [8] with later extensions. An instrumentalisation of the *Rooms* example by Craven and Sergot [1] is presented in [7]. An implementation of the simple *Estates* example in [15, pp. 610ff] is currently being developed, to demonstrate how the framework handles non-elementary norms, i.e., norms whose grounds and/or consequences are boolean combinations of simpler conditions.

Table 5. Transition Type Conditions for Extended Types (Λ mnemonic for Leave and Ω for Oppose)

	I	II	III	IV	$Prohibited_a(x, s)$ if
	-	-	-	-	-
$C^a_{2\Lambda}$	-	-	X	-	$d(X_\nu; s) \wedge \neg d(X_\nu; a(x, s))$
$C^a_{2\Omega}$	-	-	-	X	$\neg d(X_\nu; s) \wedge \neg d(X_\nu; a(x, s))$
$C^a_{4\Lambda}$	-	X	-	-	$\neg d(X_\nu; s) \wedge d(X_\nu; a(x, s))$
$C^a_{4\Omega}$	X	-	-	-	$d(X_\nu; s) \wedge d(X_\nu; a(x, s))$
C^a_{5-}	-	-	X	X	$\neg d(X_\nu; a(x, s))$
$C^a_{6\Lambda}$	-	X	X	-	$\neg(d(X_\nu; s) \leftrightarrow d(X_\nu; a(x, s)))$
$C^a_{6\Omega}$	X	-	-	X	$d(X_\nu; s) \leftrightarrow d(X_\nu; a(x, s))$
C^a_{7-}	X	X	-	-	$d(X_\nu; a(x, s))$

4 Conclusion and Future Work

Lexicons for two different extended systems of types of normative positions in the context of norm-regulated transition system situations were suggested and discussed. It was demonstrated that both understandings are natural and useful, depending on how the notion of agency (represented by the action operator Do/E_x) is understood and whether a 'system norms' or 'agent-specific norms' perspective is taken.

As already mentioned, this work aims at contributing to a typology of models for the theory of normative positions based on different assumptions. Future work includes trying to explicitly state additional assumptions regarding the logic of Do, Leave, Oppose and May that capture the meaning of these operators in a norm-regulated transition system situation context, and further investigating the formal algebraic aspects of representing normative systems under these interpretations. It would also be interesting to generalise the concept of transition system situations, and study how Lindahl's system of two-agent types of normative positions could be used to deal with simultaneous actions by two agents (including 'actions' by the environment) in this context.

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